Problem Set 3: Volume and Length

Due: September 24, 2018

Math 141, FC02

Instructor: Maxx Cho

Name: _____

You must show your work and demonstrate your understanding to receive full credit. **For volume problems, a picture must accompany your integral to justify its set-up.**

0. (10 points) Redo problem 3(b) from your first Exam. You must show every step correctly to receive full credit.

1. Consider $f(x) = x$ and $g(x) = x^2$. Let R be the region between the graphs of f and g. Let D be the solid formed by revolving R around the y-axis.

   (1) (4 points) Find the volume of D using the washer method.

   (2) (4 points) Find the volume of D using the shell method.

2. Consider $f(x) = x$ and $g(x) = x^2$. Let R be the region between the graphs of f and g. (Yes, this is the same region as in problem 1).

   (1) (4 points) Consider the solid A formed by revolving R about the line $y = 2$. Find the volume of A.

   (2) (4 points) Consider the solid B formed by revolving R about the line $x = -1$. Find the volume of B.

3. From high school geometry class, we know that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$. Follow the following steps of derive this formula.

   (1) (1 points) Consider the formula for the *circle* of radius r centered at the origin:

   $$x^2 + y^2 = r^2$$

   Solve this equation for y, then give the equation for the *upper half* of the circle only.

   (2) (4 points) Consider the region R formed by $y = 0$ and the semicircle in part (a). Revolve this region around the x-axis to obtain the sphere of radius r, then use the disc method to find its volume.

4. (5 points) Find the volume of the solid S, where its base is a disk of radius 1 and cross-sections perpendicular to the base are squares.

5. (5 points) Find the volume of a solid T, where the base of T is enclosed by the parabola $y = 1 - x^2$ and the x-axis while the cross-sections perpendicular to the base are isosceles right triangles with legs perpendicular to the y-axis.

6. (4 points) Find the volume of the region enclosed by $x = 4y^2 - y^3$ and $x = 0$ rotated about the x-axis.

7. (5 points) Let $f(x) = x^3 + \frac{1}{12x}$. Find the arc-length of the graph of f for $1 \leq x \leq 3$.